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Failure analysis of laminated plates using finite element method

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In the present investigation, laminated composite plates subjected to a concentrated load are studied in order to determine their failure behavior and also the first ply failure (FPF) load. To achieve this goal, '***Tsai-Wu***' failure criteria is implemented in a finite element code in order to predict the FPF load and the damage location. Based up on the classical laminate theory, a four-node rectangular finite element with 6 degrees of freedom at each node is formulated and used for this investigation. The numerical results obtained using the present element compare favorably with those given by the analytic approach. It is observed that the numerical results are very close to the analytical results, which demonstrates the accuracy of the present element. Finally, several parameters, such as fiber orientations, stacking sequences and boundary conditions are considered, in order to determine and understand their effects on the strength of these laminated plates.

Keywords: *Laminated composite, Tsai-Wu, failure, FPF, damage.*

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1. Introduction:

High-performance composites are commercially used in the fabrication of aircraft, automotive, and marine structures. The application of these composites transcends the boundaries of the aircraft design and has entered the field of civil engineering due to its characteristics such as high specific strength, stiffness and lightweight.

In the literature, there are many researchers who have studied the failure and damage behavior of composite plates. Reddy and Pandey [1] formulated a finite element based on the first order shear deformation theory for FPF analysis of laminated composite plates. In their work, they used maximum stress, maximum strain, Tsai-Wu and Hoffman failure criterion to analyze plates subjected to in-plane and/or bending loads. Prosti et al. [2] studied the FPF of laminated stiffened shells under different loading conditions. On the other hand, Pal and Bhattacharyya [3] studied the progressive failure analysis of laminated composite plates under transverse static loading. In addition, Moncada et al. [4] used micromechanics theories, coupled with classical lamination theory, in order to study the progressive damage in laminated composite plates using different failure theories, such as, maximum stress, maximum strain, Tsai-Hill, and Tsai-Wu failure criterion.

In the present investigation, the well-known Tsai-Wu failure criteria is used in order to predict and determine the FPF load, damage location and failure mechanism. To achieve this goal, a rectangular finite element of four-node and 6 degrees of freedom at each node is formulated, based on the classical plate theory. Several parameters, such as fiber orientations, stacking sequence and boundary conditions, are investigated to understand their effects on failure.

2. Finite element formulation:

Based on the classical laminated plate theory, the displacement field can be given using the following eq.:

$$u(x,y,z,t) = u_0(x,y,z) - zw_{0x}(x,y,t)$$
$$v(x,y,z,t) = v_0(x,y,z) - zw_{0y}(x,y,t)$$
$$w(x,y,t) = w_0(x,y,t) \quad (1)$$

The strain-displacement relation, including large deformations, can be determined as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \end{Bmatrix} + z\begin{Bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ -2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (2)$$

$$= \left\{\varepsilon_L^0 + \varepsilon_{NL}^0\right\} + z\{k\}$$

The resultant forces (N) and moments (M) are related to the mid-surface strains ε^0 and to the curvatures (k) by:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} \quad (3)$$

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n} \int_{z^k}^{z^{k+1}} \overline{Q}_{ij}^{(k)} (1, z, z^2) dz \quad (4)$$

where $[A]$, $[B]$ and $[D]$ are the extensional, coupling and bending rigidity matrix, respectively. $\overline{Q_{ij}}$ are the coefficients of the stiffness matrix of a layer in the global coordinate system (x, y, z) of a laminated plates.

3. Description of the finite element:

Based on the classical plate theory, a rectangular finite element of four-nodes with 6 degrees of freedom at each node is formulated. Two degrees of freedom in the plan (x, y) are (u, v) and the four degrees of freedom out of plan are $w, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial^2 w}{\partial x \partial y}$.

The interpolation functions N (x, y) of the coordinates are given by:

$$\delta(x,y) = \sum_{i=1}^{4} N_i(x,y)\delta_i \quad (5)$$

where $\delta(x,y)$ is the displacement vector of a given point within the element and δ_i and $N_i(x,y)$ are the nodal displacement vector and the bilinear Lagrange shape functions.

Based on the principle of minimum potential energy, the first variation allows obtaining the expression of the elementary stiffness matrix [K^e].

$$\left[K^e\right] = \iint \left\{ [B_m]^T [A][B_m] + [B_m]^T [B][B_f] + [B_f]^T [B][B_m] + [B_f]^T [D][B_f] \right\} ds \quad (6)$$

4. Failure criteria (Tsai–Wu failure criterion):

The Tsai-Wu failure criterion is a phenomenological material failure theory, widely used for anisotropic composite materials [5]. The present criteria takes account the interaction between stresses. The Tsai-Wu criterion predicts failure when the failure index in a laminate reaches 1 [6].

According to this theory, the failure criterion is expressed by:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j \geq 1 \quad i, j = 1......, 6 \quad (7)$$

So,

$$\left(\frac{1}{X_T} - \frac{1}{X_C}\right)\sigma_1 + \left(\frac{1}{Y_T} - \frac{1}{Y_C}\right)\sigma_2 + \left(\frac{1}{X_T X_C}\right)\sigma_1^2 + \left(\frac{1}{Y_T Y_C}\right)\sigma_1^2$$
$$-\frac{1}{2}\left(\sqrt{X_T X_C Y_T Y_C}\right)\sigma_1\sigma_2 + \left(\frac{\sigma_6}{X_S}\right)^2 \geq 1 \quad (8)$$

5. Numerical results and discussion:

The first study consists of a square $[0/90]_s$ laminated plate with sides a=b = 10 cm. The plates is clamped (CC) on the four sides and subjected to a concentration load (Failure load = 1410 N). The elastic properties used in this study are given in Table 1. The convergence of the displacement obtained is presented in Table 2 and shown on Fig 1.

Table. 1. Material property.

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	v_{12}	a et b (mm)	h (mm)
141000	9340	4500	0.35	100	$(0.5)_4$
X_t (MPa)	**Y_t (MPa)**	**X_C (MPa)**	**Y_C (MPa)**	**S (MPa)**	
1500	180	1000	240	150	

Table. 2. Convergence of the displacement for a clamped (CC) laminated plate subjected to a concentrated load.

Mesh	Displacement
2×2	1,9166
4×4	2,1357
6×6	2,1867
8×8	2,2003
10×10	**2,2066**
Analytical solution	**2,1750**

One can see from Table 2 and Fig.1 that, the numerical results obtained by the present element compare favorably with those obtained by the analytic solution. It is observed that the results are very close to the reference results, which demonstrates the accuracy of the present element.

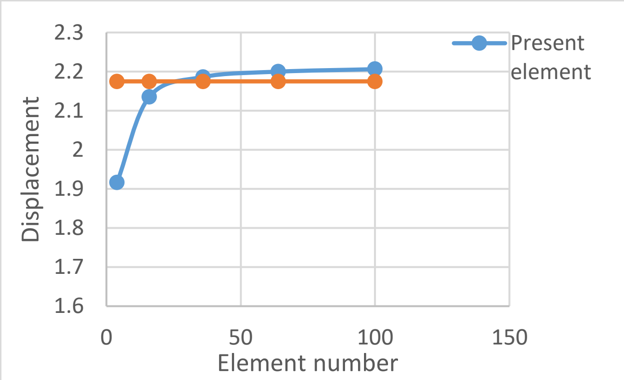

Fig.1. Convergence of the displacement for a clamped laminated plate subjected to a concentrated load.

5.1. Effect of boundary condition and stacking sequences on the FPF:

After the convergence test, the accuracy of this element was validated. Now, one can use it to analyze the influence of boundary conditions and the direct influence of fiber orientation angle on the failure load and also determine the damage location. A Tsai-Wu failure criterion is used to determine the FPF load of plates with stacking sequences $[0/\theta]_s$ under a concentrated load. Different boundary conditions are considered as given in Table 3. The results are presented in Table 4 and Figures 2 and 3. The same material property is used in this investigation.

Table .3. Boundary conditions (BC) employed in numerical tests.

	Boundary conditions	
CC	$u=0; v=0; w=0; \frac{\partial w}{\partial x}=0; \frac{\partial w}{\partial y}=0; \frac{\partial^2 w}{\partial x \partial y}=0$ $u=0; v=0; w=0; \frac{\partial w}{\partial x}=0; \frac{\partial w}{\partial y}=0; \frac{\partial^2 w}{\partial x \partial y}=0$	
SS	$u\neq0; v\neq0; w=0; \frac{\partial w}{\partial x}\neq0; \frac{\partial w}{\partial y}=0; \frac{\partial^2 w}{\partial x \partial y}\neq0$ $u\neq0; v\neq0; w=0; \frac{\partial w}{\partial x}\neq0; \frac{\partial w}{\partial y}=0; \frac{\partial^2 w}{\partial x \partial y}\neq0$	

It is observed that, for both boundary conditions, by increasing the fiber orientation angle, there is an increase in failure load, and when the stacking sequences are [0°/75°]s and [0°/90°]s, the failure loads are almost constant for both boundary conditions. The maximum failure load obtained using the present element is 1410 N for clamped plates and 1102 N for a simply supported laminated with [0°/90°]s. The first surface failure (FPF) is always the bottom side of the fourth layer.

Table. 4. The FPF strength of simply supported and clamped laminated plates with different stacking sequences.

BC	Stacking sequences	Failure load		
		Analytical Results	Present element	Location
CC	[0°/0°]s	1650	1140	Bot 4
	[0°/15°]s	1680	1160	Bot 4
	[0°/30°]s	1732	1220	Bot 4
	[0°/45°]s	1800	1300	Bot 4
	[0°/60°]s	1850	1360	Bot 4
	[0°/75°]s	1880	1400	Bot 4
	[0°/90°]s	1880	1410	Bot 4
SS	[0°/0°]s	1210	905	Bot 4
	[0°/15°]s	1230	930	Bot 4
	[0°/30°]s	1290	990	Bot 4
	[0°/45°]s	1340	1040	Bot 4
	[0°/60°]s	1370	1080	Bot 4
	[0°/75°]s	1370	1100	Bot 4
	[0°/90°]s	1370	1102	Bot 4

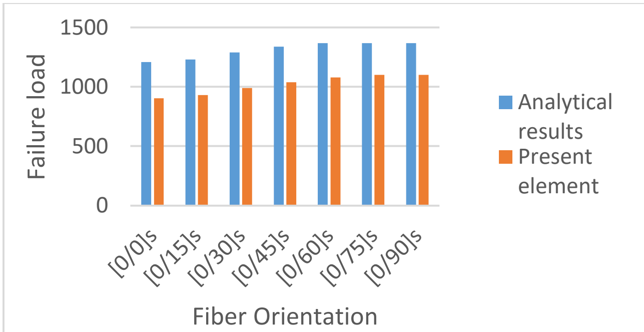

Fig.2. The FPF strength of simply supported laminated plates with different stacking sequences.

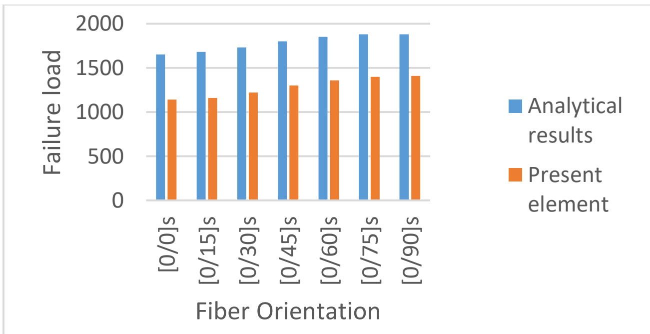

Fig.3. The FPF strength of clamped laminated plates with different stacking sequences.

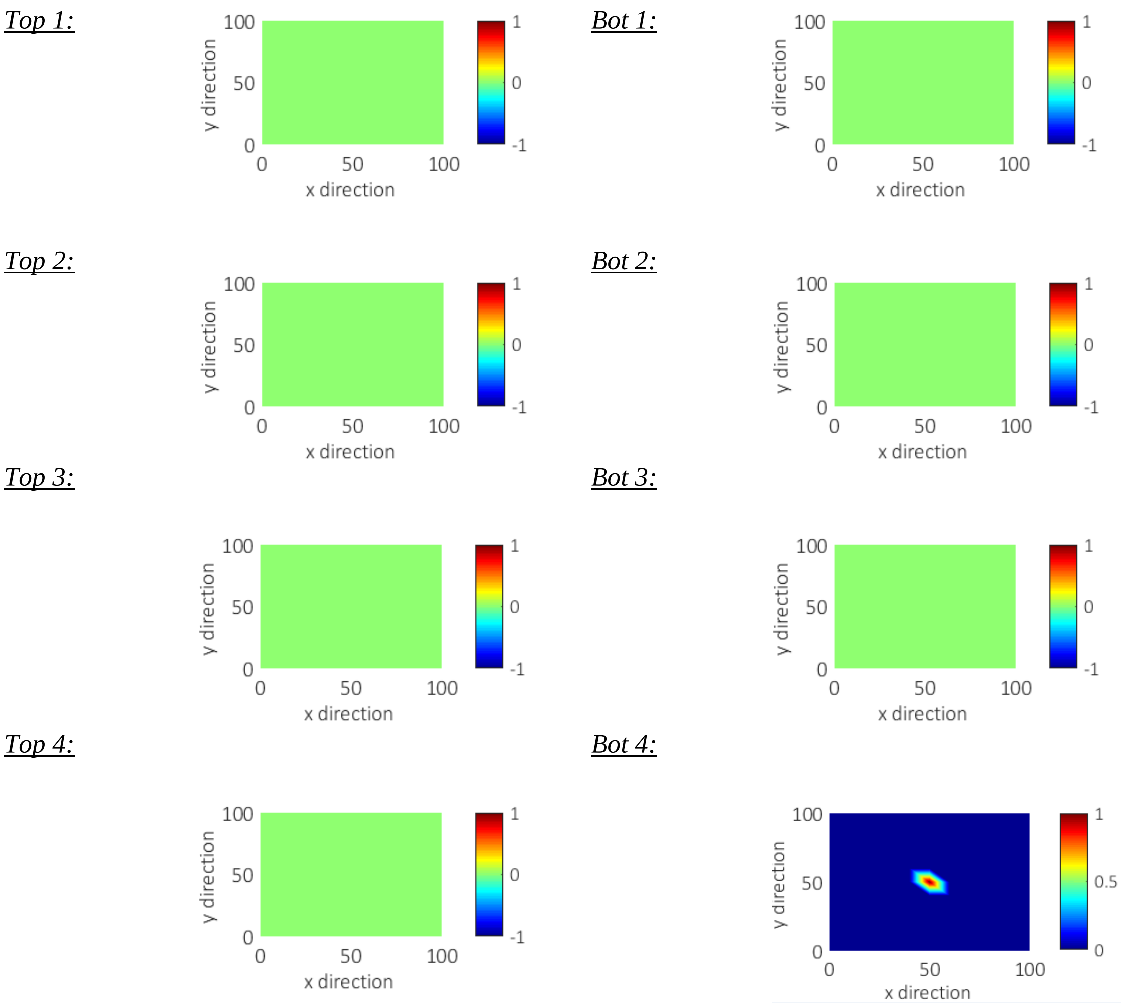

Fig.4. Damage distributions of fixed supported laminated plates [0°/90°]s under a concentrated load.

6. Conclusion:

In this paper, laminated composite plates subjected to concentrated load were studied in order to determine the FPF. The 'Tsai-Wu' failure criterion was implemented in a finite element code in order to predict the FPF load of these plates, and the damage location. Based up on the classical laminate theory, a four-node rectangular finite element with 6 degrees of freedom at each node was formulated for this purpose. The numerical results obtained by the present element compared favorably with those given by the analytic approach. It was observed that the numerical results were very close to the analytical results, which demonstrates the accuracy of the present element. Several parameters, such as fiber orientations, stacking sequence and boundary conditions were considered to see their effects on plates strength.

7. References:

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